Differentially Private Data Publishing for Arbitrarily Partitioned Data

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Abstract

Many models have been proposed to preserve data privacy for different data publishing scenarios. Among these models, ϵ-differential privacy is receiving increasing attention because it does not make assumptions about adversaries' prior knowledge and can provide a rigorous privacy guarantee. Although there are numerous proposed approaches using ϵ-differential privacy to publish centralized data of a single-party, differentially private data publishing for distributed data among multiple parties has not been studied extensively. The challenge in releasing distributed data is how to protect privacy and integrity during collaborative data integration and anonymization. In this paper, we present the first differentially private solution to anonymize data from two parties with *arbitrarily partitioned data* in a semi-honest model. We aim at satisfying two privacy requirements: (1) the collaborative anonymization should satisfy differential privacy; (2) one party cannot learn extra information about the other party's data except for the final result and the information that can be inferred from the result. To meet these privacy requirements, we propose a distributed differentially private anonymization algorithm and guarantee that each step of the algorithm satisfies the definition of secure two-party computation. In addition to the security and cost analyses, we demonstrate the utility of our algorithm

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in classification analysis.

Keywords: data publishing, ϵ-differential privacy, arbitrary partitioning, secure two-party computation, classification analysis

1. Introduction

With the development of cloud computing, data are being collected and contributed by different entities, such as shopping records by chain stores, financial data by bank branches, and census profiles by government agencies. To enable better data analysis, distributed data are often integrated before further processing. However, involving untrusted collaborators in data integration could pose a threat to the privacy of data owners. Another threat to data privacy comes from data publishing. Data owners sometimes intend to release their data to the public for various purposes, including service improvement, public competition, and academic research. For example, Netflix, an online video streaming service, opened its previous rating data for research on movie recommendations [1].

Consider a scenario of data publishing as follows. A blood collection organization and a hospital have some information about the same individuals. They hope to release their integrated data to a third-party for classification analysis, such as predicting the health status of voluntary blood donors. Considering privacy and security concerns, they do not want to violate the privacy of individuals from whom the integrated data are obtained when releasing data to the third-party. Additionally, both parties are reluctant to share other information with each other during their collaboration except for the necessary information required by data integration. In the era of big data, such a scenario has become more and more common. Thus, there is a strong motivation to develop cooperative anonymization methods on distributed data for privacy-preserving data publishing.

Some approaches have been proposed to publish distributed data while protecting privacy. Jurczyk et al. [2] and Goryczka et al. [3] addressed the problem of releasing horizontally partitioned data. Jiang et al. [4, 5] generated an anonymous version of vertically partitioned data without disclosing privacy. Kohlmayer et al. [6] focused on anonymization of data that could be distributed horizontally or vertically. Unfortunately, these works had one fundamental limitation. They adopted the k-anonymity principle [7, 8] or its extensions [9, 10] that make certain assumptions about adversaries' prior knowledge as the underlying privacy model, and they cannot provide adequate protection if the adversaries get more information beyond these assumptions [11]. Compared to the family of k-anonymity, ϵ-differential privacy [12, 13] is independent of any adversary's prior knowledge and can provide a more rigorous privacy guarantee. There are numerous differentially private approaches [14, 15, 16, 17, 18, 19] to centralized data publishing, but a very limited number of works have focused on distributed data, especially for the scenario of arbitrarily partitioned data. In this paper, we cover the gap with a differentially private solution to the problem of anonymizing arbitrarily partitioned data between two parties.

Unlike horizontally and vertically partitioned data, there is no constraint on how raw data are divided between different parties in the scenario of arbitrary partitioning. Consider a data table of n rows and d columns that is arbitrarily partitioned between two parties, P1 and P2. That is, for each row r_i ($1 \leq i \leq n$) of the data table, P1 holds a subset of p attributes (denoted by r_i^1), and P2 holds a subset of q attributes (denoted by r_i^2), such that $p + q = d$ $(0 < q < d, 0 < q < d)$, $r_i^1 \cup r_i^2 = r_i$, and $r_i^1 \cap r_i^2 = \emptyset$. For example, values of the attributes *Job* and *Age* in Table 1 are arbitrarily partitioned between two parties.

In this paper, we propose a differentially private algorithm to integrate two arbitrarily partitioned data fragments into a data table while transforming the data table into an anonymous version for classification analysis. Our work is inspired by [14], which uses the top-down specialization (TDS) technique [20] to anonymize centralized data in a differentially private manner. We extend our research scope from centralized data to distributed data with an arbitrary partitioning form. We adopt the semi-honest model, in which parties abide by defined protocols but may try to learn extra information from their received messages. Note that the anonymization of arbitrarily partitioned data among

Table 1. Data fragment arbitrarily partitioned between two parties (The first column is solely for the purpose of illustration. For the attributes *Job* and *Age*, the underlined values and the italicized values are held by P1 and P2, respectively. Both parties hold the *Class* attribute.)

No.	Job	Age	Class
1	<u>Teacher</u>	*35*	Y
2	*Clerk*	<u>25</u>	N
3	*Doctor*	<u>36</u>	Y
4	<u>Teacher</u>	*26*	N
5	<u>Clerk</u>	*25*	N
6	*Teacher*	<u>29</u>	Y
7	<u>Cook</u>	*38*	Y
8	*Clerk*	<u>23</u>	N
9	*Doctor*	<u>38</u>	Y
10	*Cook*	<u>37</u>	Y

three or more parties will be studied in our future work. The contributions of this paper are summarized as follows:

- We formally define the problem of differentially private data publishing for arbitrarily partitioned data between two parties. To the best of our knowledge, this paper is the first work that tackles this problem and addresses the challenge of securely integrating statistical information from both horizontal and vertical dimensions.
- We propose a distributed algorithm for generating anonymous data from two arbitrarily partitioned parties. To guarantee privacy, the anonymization process meets both the definitions of differential privacy and secure two-party computation; thus, during the execution of the algorithm, neither party can learn additional information on the other party's data beyond what can be inferred from the final result.
- We evaluate the utility of our distributed algorithm in classification tasks.

Experimental results show that the algorithm can provide anonymous data of good quality for classification analysis when compared to two single-party algorithms, i.e., DiffGen [14] and DiffP-C4.5 [21].

The rest of the paper is organized as follows. Preliminaries including the problem statement are presented in Section 2. The proposed algorithm is described in Section 3 and analyzed in Section 4. Experimental results are presented in Section 5, and the related work is addressed in Section 6. Section 7 concludes the paper.

2. Preliminaries

We review ϵ-differential privacy and the secure protocols used in our algorithm. Then we formally define the problem of differentially private data publishing for arbitrarily partitioned data between two parties. Table 2 provides some notations used in this paper.

Table 2. List of notations

Notations	Explanation	Notations	Explanation
P1, P2	party/data owner	ϵ	privacy budget
$\mathbb{U}$	universe	r	record
D	raw dataset	n	number of records
$\hat{D}$	neighbor dataset	d	number of attributes
D'	anonymous dataset	d_{num}	number of numerical attributes
D_1	data fragment held by P1	h	number of specializations
D_2	data fragment held by P2	l	number of digits after a decimal point
$\mathcal{M}$	mechanism	f, u	function
R	random number	Δf, Δu	global sensitivity
R_1	random number held by P1	cls	class value
R_2	random number held by P2	v, c	attribute value

2.1. Differential privacy

Let $\mathbb{U}$ represent a finite data universe, and let r represent a data record with d attributes. A dataset D consists of n records sampled from $\mathbb{U}$. Two datasets

D and $\hat{D}$ are defined as neighboring datasets if and only if either $\hat{D} = D + r$ or $D = \hat{D} + r$, where $D + r$ (or $\hat{D} + r$) denotes the dataset resulted from adding r to D (or $\hat{D}$). According to the definition of neighboring datasets, ϵ-differential privacy is defined as follows.

Definition 1 (*ϵ-differential privacy* [12])**.** A randomized mechanism $\mathcal{M}$ is differentially private if for any pair of neighboring datasets D and $\hat{D}$, and for any set of possible sanitized outputs Ω,

$$Pr[\mathcal{M}(D) \in \Omega] \leq exp(\epsilon) \times Pr[\mathcal{M}(\hat{D}) \in \Omega]. \tag{1}$$

The parameter ϵ, called privacy budget, is used for controlling the level of privacy guarantees achieved by mechanism $\mathcal{M}$. A smaller value of ϵ means a stronger privacy level. ϵ defaults to a positive number, and its selection is an open question. But the value of ϵ is usually small in the literature, such as 0.1, 0.5, and 0.8 [22].

The magnitude of added noise depends not only on the privacy budget ϵ but also on the global sensitivity of a randomized function. Global sensitivity reflects the maximum difference of outputs of a function on two neighboring datasets.

Definition 2 (*Global sensitivity* [12])**.** Given a randomized function $f : D \to \mathbb{U}$, the global sensitivity of f is

$$\Delta f = \max \|f(D) - f(\hat{D})\|_1, \tag{2}$$

for any pair of neighboring datasets D and $\hat{D}$.

The Laplace mechanism and the exponential mechanism are used extensively to achieve differential privacy.

Definition 3 (*Laplace mechanism* [23])**.** Given a dataset D, privacy budget ϵ, and a randomized function $f : D \to \mathbb{U}$, the global sensitivity of which is Δf, a mechanism $\mathcal{M}(D) = f(D) + Lap(\Delta f/\epsilon)$ satisfies ϵ-differential privacy.

Definition 4 (*Exponential mechanism* [24])**.** Given a dataset D, output range T, privacy budget ϵ, and a utility function $u : (D, T) \rightarrow \mathbb{U}$, a mechanism $\mathcal{M}$ that selects an output $t \in T$ with probability proportional to $exp(\frac{\epsilon u(D,t)}{2\Delta u})$ satisfies ϵ-differential privacy.

There are two important properties of differential privacy. They play a vital role in judging whether a mechanism satisfies differential privacy.

Property 1 (*Sequential composition* [25])**.** Let $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \cdots, \mathcal{M}_m\}$ be a set of privacy mechanisms. If each $\mathcal{M}_i$ provides ϵ_i-differential privacy and $\mathcal{M}$ is sequentially performed on a dataset, $\mathcal{M}$ will provide $(\sum_i^m \epsilon_i)$-differential privacy.

The sequential composition suggests that the privacy budget and noise accumulate linearly when a series of differentially private mechanisms is applied to the same dataset.

Property 2 (*Parallel composition* [25])**.** Let $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \cdots, \mathcal{M}_m\}$ be a set of privacy mechanisms. If each $\mathcal{M}_i$ provides ϵ_i-differential privacy on a disjointed subset of a dataset, $\mathcal{M}$ will provide $(\max\{\epsilon_1, \epsilon_2, \cdots, \epsilon_m\})$-differential privacy.

The parallel composition suggests that the degree of privacy protection depends on the maximum value of ϵ_i when a series of differentially private mechanisms is applied to different subsets of a dataset.

2.2. Secure protocols

Secure comparison protocol (SCP) [26]. This is a secure protocol for Yao's millionaires' problem [27], in which two integers are compared securely without revealing their exact values.

Random value protocol (RVP) [28]. Assume that P1 holds a number $R_1 \in \mathbb{Z}_N$, and P2 holds a number $R_2 \in \mathbb{Z}_N$. This protocol allows the two parties to collaboratively select a random number $R \in \mathbb{Z}_Q$ such that $R = (R_1 + R_2)$ *mod* $N \in [0,\, Q-1]$, where $\mathbb{Z}_N$ (or $\mathbb{Z}_Q$) is a set of non-negative integers less than N (or Q), $Q \in \mathbb{Z}_N$ is not known by either party but is shared between them, and N is the modulus associated with the cryptosystem used in the protocol.

Secure dot-product protocol (SDPP) [29]. The goal of this protocol is to securely compute the scalar product of two vectors, $V_1 = \{a_1, a_2, \cdots, a_m\}$ and $V_2 = \{b_1, b_2, \cdots, b_m\}$, held by two parties, respectively. At the end of the protocol, both parties randomly share the value of $V_1 * V_2$.

2.3. Problem statement

As depicted in Fig. 1, we focus on arbitrarily partitioned data between two parties in this paper. Suppose that two data fragments, D_1 and D_2, are held by P1 and P2, respectively. D_1 and D_2 can be integrated into a data table by matching the same instance identifiers. Columns within the data table are divided into three categories, i.e., (1) an explicit identifier that clearly identifies individuals, such as social security number, (2) a *Class* attribute that represents categories to which records belong, and (3) a set of feature attributes, which is used for predicting the class value. The explicit identifier and the *Class* attribute are assumed to be held by both P1 and P2, and the feature attributes are arbitrarily divided between them (refer Table 1). The feature attributes can consist of categorical and numerical attributes, and the two parties share the same taxonomy tree for each categorical attribute. Both parties wish to release their integrated data in an anonymous form to some data recipient for classification analysis, and during their collaboration they do not want to disclose anything more than what is required by the integration.

We further assume that there is no trusted third party who can integrate the distributed data. We adopt the semi-honest model in this paper. In the model, each party follows pre-specified protocols and honestly provides inputs to guarantee the result's correctness, but may try to learn additional information from her received messages. We focus on the privacy concerns brought by inside parties; the security and privacy issues brought by outside attackers are beyond the scope of this paper. Thus, all information exchanged between two parties is considered to be transmitted through secure authenticated channels.

Based on the above assumptions, our problem statement is defined as follows:

Definition 5 (*Differentially private data publishing for arbitrarily partitioned*

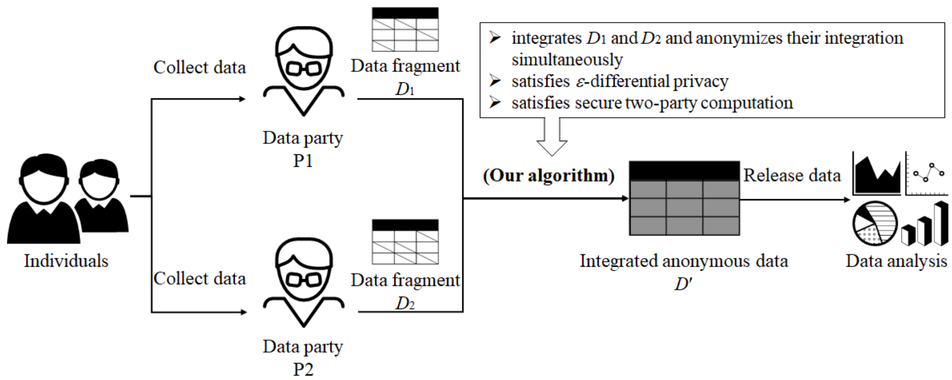


Fig. 1. Illustration of our research idea (Two parties, P1 and P2, have collected some information from the same individuals. The information is denoted by D_1 and D_2, respectively. Our research aims to integrate D_1 and D_2 into a data table and simultaneously transform the data table into an anonymous version, denoted by D'. Considering privacy and security concerns, we guarantee that our proposed algorithm satisfies ϵ-differential privacy and secure two-party computation. D' will be released to the public/third-party for data analysis.)

data between two parties). Given a data fragment D_1 held by P1, another data fragment D_2 held by P2, and privacy budget ϵ, D_1 and D_2 can be integrated into a data table with n records and d feature attributes. For each row r_i $(1 \leq i \leq n)$ of the data table, P1 holds a subset of p attributes (denoted by r_i^1), and P2 holds a subset of q attributes (denoted by r_i^2), such that $p + q = d$ $(0 < p < d, 0 < q < d)$, $r_i^1 \cup r_i^2 = r_i$ and $r_i^1 \cap r_i^2 = \emptyset$. The problem of differentially private data publishing for arbitrarily partitioned data is to generate an integrated anonymous version of D_1 and D_2 such that the generation algorithm (1) satisfies ϵ-differential privacy and (2) follows the definition of secure two-party computation in the semi-honest model.

3. Proposed algorithm

In this section, we first present an overview of our differentially private algorithm for anonymizing arbitrarily partitioned data between two parties. We then elaborate key steps of the algorithm.

3.1. Overview

Our distributed algorithm is modified significantly based on the top-down specialization (TDS) technique [20]. As stated in [20], the specialization starts with the most general state and goes down iteratively by replacing some values with more specific values until reaching the predefined number of specializations. A specialization, denoted by $v \rightarrow Children(v)$, replaces a parent value v with its directly connected child values $Children(v)$ according to the corresponding taxonomy tree. For instance, in Fig. 3, *Children(ANY_JOB)* = {*Professional*, *Worker*}, and *Children*([1, 99)) = {[1, 26), [26, 99)}. We also refer the parent value that can be replaced by its directly connected child values to as "cut" in the following.

Example 1. Fig. 2 shows a process of the TDS technique on Table 1 (data of Table 1 is now treated as centralized data). At first, each value is generalized to the topmost value of its corresponding taxonomy tree presented in Fig. 3, and the initial $\cup Cut$ is {*ANY_JOB*, [1, 99)}. If the *ANY_JOB* cut is selected to split downwards, the root of the partition tree in Fig. 2 will have two new child nodes because of *ANY_JOB* → {*Professional*, *Worker*}, and the current $\cup Cut$ will be updated to {*Professional*, *Worker*, [1, 99)}, each element of which can continue to be selected and split. □

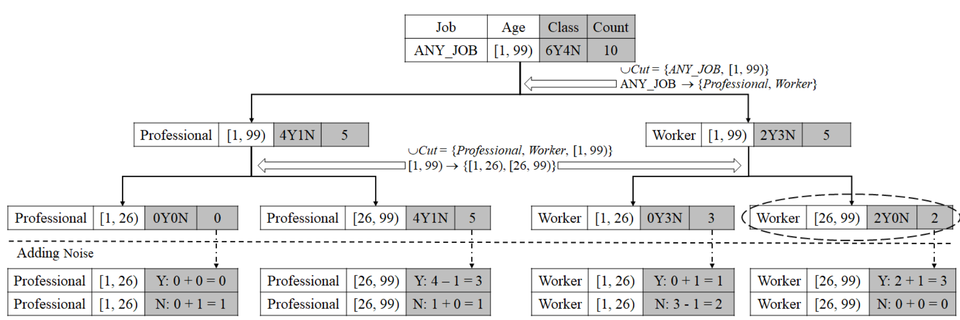


Fig. 2. Example of a partition tree

The differentially private TDS on distributed data is similar to that on centralized data. The difference is that for distributed data, the statistical informa-

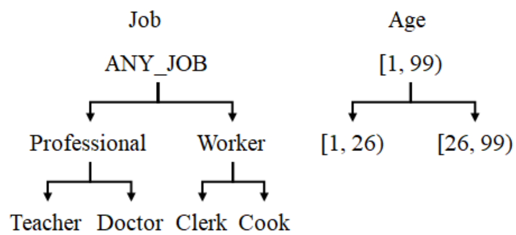


Fig. 3. Taxonomy trees of the attributes *Job* and *Age*

tion required by the TDS should be integrated securely from different parties. We present our Arbitrarily Distributed Differentially Private anonymization algorithm (ArbDistDP) as shown in Algorithm 1. Two parties can run ArbDistDP separately; however, Lines 4, 6, 7, 11, and 12 of ArbDistDP must be executed collaboratively (while other lines can be done by a single party). Each of the two parties maintains her own $\cup Cut$ during the execution of ArbDistDP and obtains the final integrated anonymous data same as that obtained by the other party.

ArbDistDP consists of two phases, i.e., generalizing raw data in a top-down manner, and adding noise to the generalized result. We adopt the uniform allocation rule to allocate the privacy budget ϵ for each phase. Namely, one half of ϵ is allocated for the first phase while the other half is for the second phase. Besides, in the first phase each differentially private step consumes the same amount of privacy budget, denoted by ϵ'. More specifically, an initial split value is selected for each of d_{num} numerical attributes in Line 4, Line 7 is executed h times, and Line 11 is executed at most h times; thus, the privacy budget ϵ' equal to $\frac{\epsilon}{2(d_{num}+2h)}$ is allocated for Lines 4, 7, and 11, respectively.

To ensure that ArbDistDP meets the security requirements of ϵ-differential privacy and secure two-party computation, the key is to make each specialization differentially private and secure. The key steps of ArbDistDP include (1) selection of split values, (2) calculation of utility scores, (3) selection of cuts, and (4) adding noise. Although step (1) is presented before steps (2) and (3) in ArbDistDP, steps (2) and (3) are described before step (1) in the following because step (1) is dependent on them. These steps also are the key to addressing the challenge of securely integrating statistical information from both horizontal and vertical dimensions.

Algorithm 1: ArbDistDP (executed by P1; similar to P2)

Input: D_1: data fragment held by P1 // input D_2 if the algorithm is executed by P2
ϵ: privacy budget
h: number of specializations
Output: D': integrated generalized data
// Phase 1: Generalizing raw data in a top-down manner.
$D' \leftarrow$ {each value in D_1 is generalized to the topmost value};
$\cup Cut \leftarrow$ {all topmost values};
$\epsilon' \leftarrow \frac{\epsilon}{2(d_{num}+2h)}$; // d_{num} is the number of numerical attributes.
select a split value for each numerical attribute by Algorithm 4; // Refer Section 3.4.
for $i \leftarrow 1$ *to* h **do**
    compute the utility score of each (new) cut in $\cup Cut$ by Algorithm 2; // Refer Section 3.2.
    select a cut v from $\cup Cut$ by Algorithm 3; // Refer Section 3.3.
    specialize v in D' according to $v \rightarrow Children(v)$; // v is an attribute value in D'.
    update $\cup Cut$ by replacing v with $Children(v)$; // v also is an element of $\cup Cut$.
    if v *is numerical* **then**
        select a split value for v by Algorithm 4; // Refer Section 3.4.
// Phase 2: Adding noise to the generalized data.
add noise to the number of records in D' by Algorithm 5; // Refer Section 3.5.
return D';

Example 2. Fig. 4 shows an example of the data processing of ArbDistDP on Table 1. Note that the penultimate data table shown in Fig. 4 can be generalized further according to users' requirements. □

3.2. Calculation of utility scores

Since we consider the classification analysis in this paper, we adopt the *Max* operator [21] as our utility measurement. The operator evaluates attribute values in terms of frequencies of class values and helps to minimize the probability of misclassification. The *Max* utility of an attribute value, v, in a dataset, D,

D_1:

No.	Job	Age	Class
1	Teacher		Y
2		25	N
3		36	Y
4	Teacher		N
5	Clerk		N
6		29	Y
7	Cook		Y
8		23	N
9		38	Y
10		37	Y

Line 1

Job	Age
ANY_JOB	[1, 99)

Line 2: $\cup Cut = \{ANY\_JOB, [1, 99)\}$

Line 7: if *ANY_JOB* is selected to split

Line 8

Job	Age
Professional	[1, 99)
Worker	[1, 99)

Line 9: $\cup Cut = \{Professional, Worker, [1, 99)\}$

Line 7: if [1, 99) is selected to split

Line 8

Job	Age
Professional	[1, 26)
Professional	[26, 99)
Worker	[1, 26)
Worker	[26, 99)

Line 9: $\cup Cut = \{Professional, Worker, [1, 26), [26, 99)\}$

Line 12

D':

Job	Age	Class	Noisy Count
Professional	[1, 26)	Y	0
Professional	[1, 26)	N	1
Professional	[26, 99)	Y	3
Professional	[26, 99)	N	1
Worker	[1, 26)	Y	1
Worker	[1, 26)	N	2
Worker	[26, 99)	Y	3
Worker	[26, 99)	N	0

Fig. 4. Data processing of ArbDistDP on one fragment of Table 1

is defined as follows:

$$Max(D, v) = \sum_{c \in Children(v)} max(|D_c^{cls}|), \tag{3}$$

where $Children(v)$ is a set of directly connected child values of v, and $|D_c^{cls}|$ is the number of records in D with generalized value c and class value cls. The global sensitivity of $Max(D, v)$ is 1 because the value of $Max(D, v)$ varies at most by 1 no matter whether adding or removing any single record.

Example 3. The initial $\cup Cut$ of Table 1 is $\{ANY\_JOB$, [1, 99)$\}$. According to Equation (3), the utility score of *ANY_JOB* is 7, where $7 = |D^Y_{Professional}| + |D^N_{Worker}| = 4 + 3$, and the utility score of [1, 99) is 9, where $9 = |D^Y_{[1,26)}| + |D^N_{[26,99)}| = 3 + 6$. □

Algorithm 2 depicts the process of securely computing a cut's utility score. For the convenience of description, the case of binary classification is considered in this algorithm. For the case of multi-class, Algorithm 2 can be extended easily by computing $|D_c^{cls_i}| - |D_c^{cls_j}|$ for each pair of class values (cls_i, cls_j) with respect to each value $c \in Children(v)$. In Algorithm 2, Lines 2-3 are to determine the class value that has the highest frequency corresponding to each value c. Once the class value is determined for value c, P1 and P2 collaboratively compute the number of records that have attribute value c and class value cls. In order to protect the privacy of intermediate results, both parties generate a random number to perturb their real number of records and send the perturbed value to each other (Lines 4-9). At the end of each round, P1 holds R_1 and $|D_c^{cls}| - R_2$, while P2 holds R_2 and $|D_c^{cls}| - R_1$. The sum of the four values is equal to $max(|D_c^{cls}|)$. At the end of Algorithm 2, each party randomly shares the utility score of cut v.

Example 4. P1 and P2 collaboratively compute the utility score of *ANY_JOB* using Algorithm 2. According to Equation (3), they first ascertain the class value that has the highest frequency corresponding to each child value of *ANY_JOB*.

For *ANY_JOB* $\rightarrow$ *Professional*,

Algorithm 2: Securely computing the utility score of a cut

Input: v: cut

Output: P1 holds one part of v's utility score, and P2 holds the other part.

for $\forall c \in Children(v)$ **do**

  P1 calculates $d_1 \leftarrow |D_c^Y| - |D_c^N|$, and P2 calculates $d_2 \leftarrow |D_c^N| - |D_c^Y|$; `// The symbols` Y `and` N `denote the class values.`

  P1 and P2 run SCP to compare d_1 and d_2;

  P1 generates a random number R_1, and P2 generates a random number R_2;

  if $d_1 > d_2$ **then**

    P1 calculates $t_1 \leftarrow |D_c^Y| - R_1$, and P2 calculates $t_2 \leftarrow |D_c^Y| - R_2$;

  else

    P1 calculates $t_1 \leftarrow |D_c^N| - R_1$, and P2 calculates $t_2 \leftarrow |D_c^N| - R_2$;

  P1 sends t_1 to P2, and P2 sends t_2 to P1;

P1/P2 calculates the sum of all random numbers she generated and all values she received; `// The sum is part of` v`'s utility score held by P1/P2.`

P1: $|D^Y_{Professional}| = 1$, $|D^N_{Professional}| = 1$

$d_1 = |D^Y_{Professional}| - |D^N_{Professional}| = 1 - 1 = 0$

P2: $|D^N_{Professional}| = 0$, $|D^Y_{Professional}| = 3$

$d_2 = |D^N_{Professional}| - |D^Y_{Professional}| = 0 - 3 = -3$

As $d_1 > d_2$, the value *Professional* corresponds to the class value Y with the higher frequency compared to the class value N. P1 and P2 separately generate a random number to perturb their real number of records with attribute value *Professional* and class value Y.

P1: $R_1 = 1$, $|D^Y_{Professional}| - R_1 = 1 - 1 = 0$

P2: $R_2 = 2$, $|D^Y_{Professional}| - R_2 = 3 - 2 = 1$

P1 sends the perturbed value 0 to P2, and P2 sends the perturbed value 1 to P1. P1 calculates the sum of the random number she generated and the value she received, which is $1+1 = 2$. Similarly, P2 gets the sum 2, which is $2+0 = 2$. Thus, $|D^Y_{Professional}| = 4$, of which P1 and P2 each hold a half separately.

After the similar calculation of *ANY_JOB* $\rightarrow$ *Worker*, both parties randomly share the entire utility score of *ANY_JOB*. $\square$

3.3. Selection of cuts

After obtaining each cut's utility score, P1 and P2 collaboratively select a cut from the current $\cup Cut$. The exponential mechanism is adopted in this step because it is designed for discrete alternatives. According to Definition 4, the exponential mechanism selects a candidate with the probability proportional to its utility score. We extend the distributed exponential mechanism DistExp proposed in [30]. The general idea of DistExp is to divide the range [0, $\sum_i exp(\frac{\epsilon u_i}{2\Delta u})$] into multiple segments, each corresponding to a cut and having a sub-interval of length equal to $exp(\frac{\epsilon u_i}{2\Delta u})$, then to uniformly select a random number from the range [0, $\sum_i exp(\frac{\epsilon u_i}{2\Delta u})$]. The segment in which the random number falls corresponds to the winner cut. For example, in Fig. 5 the probability of selecting a random value from Segment 1, Segment 2, Segment 3, and Segment 4 of the range [0, 1] is 20%, 30%, 10%, and 40%, respectively.

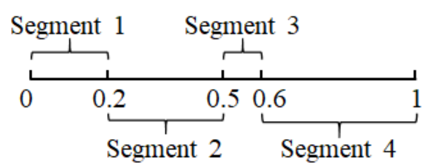


Fig. 5. Example of a segmentation of the range [0, 1]

The challenge facing us is the randomness share of the utility score of each cut. To tackle this problem, we leverage one property of the exponential function, i.e., $exp(\frac{\epsilon u}{2\Delta u}) = exp(\frac{\epsilon(u_1+u_2)}{2\Delta u}) = exp(\frac{\epsilon u_1}{2\Delta u}) \times exp(\frac{\epsilon u_2}{2\Delta u})$, where u is the entire utility score of a cut, u_1 and u_2 are held by P1 and P2 respectively, and $u_1 + u_2 = u$. Then we let P1 and P2 use SDPP to compute the product of $exp(\frac{\epsilon u_1}{2\Delta u})$ and $exp(\frac{\epsilon u_2}{2\Delta u})$. In fact, this calculation is treated as converting the values of $exp(\frac{\epsilon u_1}{2\Delta u})$ and $exp(\frac{\epsilon u_2}{2\Delta u})$ into two vectors and using SDPP to compute the inner product of the two vectors.

The two parties run RVP to collaboratively select a random number from the range [0, $\sum_i exp(\frac{\epsilon u_i}{2\Delta u})$]. The RVP works only for integers, but $exp(\frac{\epsilon u_i}{2\Delta u})$ may be a floating-point number. In this case, before computing the product we scale floating-point numbers by taking their floor values of $exp(\frac{\epsilon u_i}{2\Delta u}) \times 10^l$, where l is the number of the considered digits after the decimal point, which can be predefined by the parties. Such a scaling is a common method of dealing with floating-point numbers in cryptography [31]. We present the process of securely selecting a cut in Algorithm 3.

Algorithm 3: Securely selecting a cut

Input: current cuts $\cup Cut$ and their utility scores

Output: winner cut

```
for i ← 1 to |∪Cut| do
    P1 calculates s_i1 ← exp(εu_i1 / 2Δu), and P2 calculates s_i2 ← exp(εu_i2 / 2Δu);
    P1 and P2 run SDPP to compute the product of s_i1 and s_i2; one part of the product
      (denoted by t_i1) is held by P1, while the other part (denoted by t_i2) is held by P2;
P1 calculates T_1 ← Σ t_i1, and P2 calculates T_2 ← Σ t_i2;
P1 and P2 run RVP to securely select a random number R from the range [0, T_1 + T_2]; P1
  gets R_1, and P2 gets R_2, where R_1 + R_2 = R;
for i ← 1 to |∪Cut| do
    P1 calculates C_1 ← t_i1 − R_1, and P2 calculates C_2 ← R_2 − t_i2;
    P1 and P2 run SCP to compare C_1 and C_2;
    if C_1 > C_2 then
        // This case means t_i1 + t_i2 > R.
        return the i^th cut of ∪Cut;
```

Example 5. Continued from Example 3. Consider the *ANY_JOB* cut, the utility score of which is equal to 7. Suppose that P1 holds one part of the utility score, $u_1 = 3$, and P2 holds the other part, $u_2 = 4$. Set $\epsilon = 0.5$ and $l = 1$.

P1: $s_1^{ANY\_JOB} = exp(\frac{0.5\times 3}{2\times 1}) \approx 2.1170$

P2: $s_2^{ANY\_JOB} = exp(\frac{0.5\times 4}{2\times 1}) \approx 2.7183$

After scaling, P1 ends up with the value 21, and P2 ends up with the value 27. The product of 21 and 27 is equal to 567, and suppose that P1 holds a value 186 while P2 holds a value 381, where 186 + 381 = 567.

For the [1, 99) cut, the utility score of which is equal to 9, suppose that P1 holds one part of the utility score, $u_1 = 7$, and P2 holds the other part, $u_2 = 2$.

P1: $s_1^{[1,99)} = exp(\frac{0.5\times 7}{2\times 1}) \approx 5.7546$

P1: $s_2^{[1,99)} = exp(\frac{0.5\times 2}{2\times 1}) \approx 1.6487$

P1 ends up with the value 57 and P2 ends up with the value 16 after scaling. The product of 57 and 16 is equal to 912, and suppose that P1 holds a value 405, while P2 holds a value 507, where $405 + 507 = 912$.

The parties collaboratively select a random number from the range [0, 1479] using the RVP, where $1479 = 567 + 912$. Suppose that the random number R is 263, for which P1 holds a value 95, and P2 holds a value 168. For the first candidate ANY_JOB, P1 calculates $C_1 = 186 - 95 = 91$, and P2 calculates $C_2 = 168 - 381 = -213$. Because $91 > -213$, the random number R lies in the range [0, 567), which means that ANY_JOB is selected as the winner cut. $\square$

3.4. Selection of split values

We assume that a taxonomy tree is provided for each categorical attribute, and both parties know it as their prior knowledge. Thus, categorical cuts are split downwards directly according to their corresponding taxonomy trees.

As mentioned in [20], there is no need to provide taxonomy trees for numerical attributes. If a numerical cut is selected to split, its corresponding taxonomy tree can be grown dynamically by searching for a split value for the numerical cut. A split value should not be picked randomly because the probability of choosing the same value from a dataset not containing this value is 0. This means that the selection on a split value for a numerical attribute is probabilistic. Our selection strategy is to split the numerical domain into two sub-intervals by each numerical value and calculate the utility scores of these sub-intervals. After all the numerical values in the domain have been processed, Algorithm 3 is adopted to select a value as the split value according to the utility scores of its corresponding sub-intervals.

Algorithm 4 depicts the process of securely selecting a split value for a numerical attribute. In Lines 1-2 of the algorithm, P1 and P2 separately sort their values of the target numerical attribute in ascending order and generate cursors to point to the first element of their sorted lists. In Lines 3-16, both parties securely compare the values pointed to by cur_1 and cur_2. They know the result of each secure comparison, and the party who holds the smaller number sends a variable named *tmp* to the other party (refer Lines 8, 11, and 18 of Algorithm 4). The variable *tmp* is a signal that tells the receiver that the smaller value held by the sender will be treated as a split value temporarily, and it will split the domain into two sub-intervals, i.e., [*min*, *tmp*) and [*tmp*, *max*). However, the receiver does not know the real value of *tmp*. In Lines 16 and 20, P1 and P2 treat intervals [*min*, *tmp*) and [*tmp*, *max*) as two cuts and use Algorithm 2 to compute the utility scores of them. Note that a party can finish the calculation according to the cursor even though the real value of *tmp* is not known to her. More specifically, for computing the utility score of interval [*min*, *tmp*), both parties only need to focus on all the values before their cursors, and for computing the utility score of interval [*tmp*, *max*), they only need to focus on all the values after their cursors (including the value pointed to by the cursors). In Line 21, P1 and P2 finally select a value as the split value for the input numerical attribute using Algorithm 3.

Example 6. Consider the *Age* attribute in Table 1. P1 has values of 25, 36, 29, 23, 38, and 37, and P2 has values of 35, 26, 25, and 38.

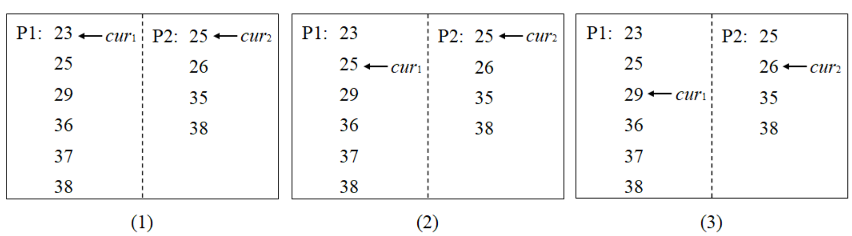


Fig. 6. Cursor movements of the *Age* attribute

P1 and P2 separately sort their values and generate a cursor to point to the first element of the sorted list. At first, cur_1 points to 23, and cur_2 points to 25 (refer Fig. 6(1)). Because $23 < 25$, P1 initializes a variable, named *tmp*, to represent 23 and sends the variable name to P2. Because P1 and P2 run SCP to compare two numbers, P2 does not know the real value of *tmp*. The two parties compute the utility score of intervals $[1, tmp)$ and $[tmp, 99)$. Although P2 does not know the value of *tmp*, she knows that cursor cur_2 points to a larger value

Algorithm 4: Securely selecting a split value for a numerical attribute

Input: A_{num}: numerical attribute

L_1: list of values of A_{num} held by P1

L_2: list of values of A_{num} held by P2

Output: v: split value for A_{num}

$L'_1 \leftarrow$ P1 sorts L_1 in ascending order, $L'_2 \leftarrow$ P2 sorts L_2 in ascending order;

$cur_1 \leftarrow$ location of the first element of L'_1, $cur_2 \leftarrow$ location of the first element of L'_2;

while $cur_1 \neq NULL$ && $cur_2 \neq NULL$ **do**

  $v_1 \leftarrow$ value pointed by cur_1, $v_2 \leftarrow$ value pointed by cur_2;

  P1 and P2 run SCP to compare v_1 and v_2;

  if $v_1 < v_2$ **then**

    P1 initializes a variable named tmp to represent v_1 and sends tmp to P2; P2 receives only a variable name but does not know its real value.

    $cur_1 \leftarrow cur_1 + 1$; `// Move` cur_1 `to the next position.`

  else if $v_2 < v_1$ **then**

    P2 initializes a variable named tmp to represent v_2 and sends tmp to P1; P1 receives only a variable name but does not know its real value.

    $cur_2 \leftarrow cur_2 + 1$;

  else

    $tmp \leftarrow v_1$; `// or` $tmp \leftarrow v_2$

    $cur_1 \leftarrow cur_1 + 1$, $cur_2 \leftarrow cur_2 + 1$;

  $s_1 \leftarrow$ P1 and P2 run Algorithm 2 to compute the utility score of interval $[min, tmp)$, $s_2 \leftarrow$ P1 and P2 run Algorithm 2 to compute the utility score of interval $[tmp, max)$; `// The utility score of` tmp `is equal to` $s_1 + s_2$ `and is randomly shared between P1 and P2.`

while $cur_i \neq NULL$ $(i = 1 or\ 2)$ **do**

  Pi initializes a variable named tmp to represent v_i and sends tmp to the other party; the receiver receives only a variable name but does not know its real value.

  $cur_i \leftarrow cur_i + 1$;

  $s_1 \leftarrow$ P1 and P2 run Algorithm 2 to compute the utility score of interval $[min, tmp)$, $s_2 \leftarrow$ P1 and P2 run Algorithm 2 to compute the utility score of interval $[tmp, max)$;

$v \leftarrow$ P1 and P2 use Algorithm 3 to select a split value;

return v;

than tmp. Thus, P2 concludes that interval [1, tmp) includes values before cur_2, and the interval [tmp, 99) includes the values after cur_2 (including the value pointed to by cur_2). After collaboratively getting utility scores of intervals [1, tmp) and [tmp, 99), P1 moves cur_1 to the next position (refer Fig. 6(2)).

As shown in Fig. 6(2), both cur_1 and cur_2 point to 25. Because $25 = 25$, P1 and P2 cooperatively compute the utility score of intervals [1, tmp) and [tmp, 99), where the value of tmp is equal to 25. After getting utility scores of intervals [1, 25) and [25, 99), both parties move their cursors to the next position respectively (refer Fig. 6(3)).

The parties continue the above process until both cur_1 and cur_2 reach the end of the sorted lists. P1 and P2 finally select a split value using Algorithm 3 according to the sum of the utility scores of the corresponding sub-intervals. □

3.5. Adding noise

After the raw data are generalized to a specific level, it is necessary to add noise to them. This is because for a different dataset, the number of records in each leaf node of the partition tree may be different and publishing the real number could violate differential privacy. We use the terms "leaf node" and "equivalence group" interchangeably in the following. This difference can be offset easily by adding noise to the number of records in each equivalence group. We adopt the secure mechanism used in [30] to add noise. In this mechanism, both parties first calculate the number of records in each group and add noise to the number using the Laplace mechanism. Algorithm 5 depicts the secure mechanism. Next, we elaborate key steps of Algorithm 5.

Calculation of the real number of records. In Line 3 of Algorithm 5, P1 and P2 separately generate a binary vector according to their own data fragment. If P1/P2 holds a record that contains the i^{th} attribute value of the current leaf node, the i^{th} element of the binary vector is set to 1; otherwise, it is set to 0. In Lines 4-6 of Algorithm 5, P1 and P2 use SDPP to securely calculate the inner product of their binary vectors, which is equal to the number of records in the

Algorithm 5: Generating the final integrated data

Input: partition tree maintained by P1/P2 `// Note that the tree held by P1 is the same as the tree held by P2.`

Output: D': integrated anonymous data

P1 generates a cryptographic key pair (PK, SK) of a homomorphic encryption scheme and sends PK to P2.

for $\forall$*node* $\in$ {*leaf nodes of the partition tree*} **do**

> `// compute the number of records in` *node*
>
> P1 generates a binary vector $V_1 \leftarrow (a_1, a_2, \cdots, a_n)$, where $a_i = 1 (1 \leq i \leq n)$ P1 has the records that contains the i^{th} attribute value of *node*; otherwise, $a_i \leftarrow 0$; P2 generates a binary vector $V_2 \leftarrow (b_1, b_2, \cdots, b_n)$, where $b_i = 1 (1 \leq i \leq n)$ if P2 has the records that contains the i^{th} attribute value of *node*; otherwise, $b_i \leftarrow 0$;
>
> P1 encrypts V_1 with PK and sends the encrypted vector $(e(a_1), e(a_2), \cdots, e(a_n))$ to P2;
>
> P2 generates a random number C_2, encrypts it with PK, and computes $P = e(R) \cdot \prod_{i=1}^{n} y_i$, where $y_i = e(a_i)$ if $b_i = 1$, and $y_i = 1$ if $b_i = 0$; P2 sends P to P1;
>
> P1 uses SK to decrypt the value received from P2 and gets the result, C_1;
>
> `// add noise to the number of records`
>
> P1 randomly samples two variables, t_1 and t_2, from Gaussian distribution $\mathcal{N}(0, \sqrt{1/\epsilon})$ and computes $X_1 \leftarrow C_1 + t_1^2 - t_2^2$, and then sends X_1 to P2; P2 randomly samples two variables, t_3 and t_4, from Gaussian distribution $\mathcal{N}(0, \sqrt{1/\epsilon})$ and computes $X_2 \leftarrow C_2 + t_3^2 - t_4^2$, and then sends X_2 to P1;
>
> $X \leftarrow X_1 + X_2$;
>
> $D' \leftarrow D' \cup$ {the generalized record in *node*, of which the noisy number of records is X};

return D';

leaf node.

Example 7. Consider the number of records in the bottom right leaf node, i.e., <*Worker*, [26, 99)>, in Fig. 2 is required. The leaf node contains records whose job can be generalized to *Worker* and age can be generalized to the range [26, 99). As detailed in Table 3, P1 generates the binary vector $V_1 = \{0, 0, 1, 0, 1, 1, 1, 0, 1, 1\}$ while P2 generates the binary vector $V_2 = \{1, 1, 0, 1, 0, 0, 1, 1, 0, 1\}$. P1 and P2 use SDPP to securely compute the inner product of V_1 and V_2 such that

$$V_1 \cdot V_2 = \sum_{i=1}^{10} Z_1(i) \times Z_2(i) = 0+0+0+0+0+0+1+0+0+1 = 2$$

At the end of SDPP, the parties have random shares of the result $V_1 \cdot V_2$, which is equal to the number of records in leaf node <*Worker*, [26, 99)>. $\square$

Table 3. Binary vectors generated by two parties according to leaf node <*Worker*, [26, 99)>

No.	P1			P2		
	Job	Age	$V_1(i)$	Job	Age	$V_2(i)$
1	Professional		0		[26, 99)	1
2		[1, 26)	0	Worker		1
3		[26, 99)	1	Professional		0
4	Professional		0		[26, 99)	1
5	Worker		1		[1, 26)	0
6		[26, 99)	1	Professional		0
7	Worker		1		[26, 99)	1
8		[1, 26)	0	Worker		1
9		[26, 99)	1	Professional		0
10		[26, 99)	1	Worker		1

Lemma 1. [32, 30] Given four Gaussian random variables, i.e., $t_i \sim \mathcal{N}(0, \lambda)$ for $i \in \{1, 2, 3, 4\}$, the random variable $Lap(2\lambda^2)$ is equal to $t_1^2 + t_2^2 - t_3^2 - t_4^2$.

Calculation of the noisy number of records. According to Lemma 1, a random variable sampled from $Lap(2\lambda^2)$ is equal to the linear combination of four random variables sampled from $\mathcal{N}(0, \lambda)$. As mentioned before, $\epsilon/2$ is used for obtaining the noisy number of records. It is easy to make an equation, $2\lambda^2 = 2/\epsilon$, and get the value of λ, which is equal to $\sqrt{1/\epsilon}$. Thus, to calculate the noisy number, P1 only needs to calculate $X_1 \leftarrow C_1 + t_1^2 - t_3^2$, where C_1 is the random share of $V_1 \cdot V_2$ held by P1, and t_1 and t_3 are sampled from Gaussian distribution $\mathcal{N}(0, \sqrt{1/\epsilon})$ by P1; similarity, P2 only needs to calculate $X_2 \leftarrow C_2 + t_2^2 - t_4^2$, where C_2 is the random share of $V_1 \cdot V_2$ held by P2, and t_2 and t_4 are sampled from Gaussian distribution $\mathcal{N}(0, \sqrt{1/\epsilon})$ by P2. Then P1 sends X_1 to P2 while P2 sends $X2$ to P1 (refer Line 7 of Algorithm 5). The noisy number of records in each leaf node, denoted by X, is calculated as follows:

$$\begin{aligned} X &= C + Lap(2/\epsilon) \\ &= X_1 + X_2 \\ &= C_1 + t_1^2 - t_3^2 + C_2 + t_2^2 - t_4^2. \end{aligned} \tag{4}$$

, where C is the real number, $Lap(2/\epsilon)$ is the added noise, X_1 is held by P1, and X_2 is held by P2.

Example 8. Noise is added to the number of records in each leaf node of the partition tree in Fig. 2 (refer the dotted arrows). □

4. Analysis of the algorithm

We analyze the correctness, security, and complexity of ArbDistDP in this section.

Theorem 1 (Correctness)**.** *ArbDistDP satisfies ϵ-differential privacy.*

Proof. Instead of developing new mechanisms, we use two existing mechanisms, i.e., the Laplace mechanism and the exponential mechanism, to design ArbDistDP. So, we prove Theorem 1 by proofing all differentially private operations in the algorithm following the two mechanisms. We first prove that all sub-algorithms used in ArbDistDP satisfy differential privacy.

– Algorithm 2 calculates the utility score of each cut that may be selected to split downwards. According to [24], the utility scores of candidates required by the exponential mechanism are based on the real counts from the raw data. Thus, Algorithm 2 does not violate differential privacy.

– Algorithm 3 selects a cut v from $\cup Cut$ with probability proportional to $exp(\frac{\epsilon u_i}{2\Delta u})$. Two parties separately compute part of the utility score $exp(\frac{\epsilon u_i}{2\Delta u})$ of each cut. They then build a range $[0, \sum_{i=1}^{n} exp(\frac{\epsilon u_i}{2\Delta u})]$ and partition the range into sub-intervals, each of which has a length equal to $exp(\frac{\epsilon u_i}{2\Delta u})$. A cut is selected according to a random value that lies uniformly in the range $[0,\ \sum_{i=1}^{n} exp(\frac{\epsilon u_i}{2\Delta u})]$, thus the probability of choosing any cut is equal to $\frac{exp(\frac{\epsilon u_i}{2\Delta u})}{\sum_{i=1}^{n} exp(\frac{\epsilon u_i}{2\Delta u})}$. Therefore, Algorithm 3 implements the exponential mechanism and satisfies differential privacy.

– Lines 1-20 of Algorithm 4 also calculate the utility score of each candidate that may be selected as a split value. Line 21 of Algorithm 4 uses the exponential mechanism to do the selection. Thus, Algorithm 4 does not violate differential privacy.

– Algorithm 5 uses the Laplace mechanism to output the noisy number of records in each leaf node of the partition tree, where the noises are sampled from $Lap(2/\epsilon)$. Thus, Algorithm 5 satisfies differential privacy.

Next, we prove that each step of ArbDistDP satisfies differential privacy.

– Line 4 of ArbDistDP selects an initial split value for each numerical attribute using Algorithm 4. The privacy budget costed by each exponential mechanism is ϵ', so the step guarantees $\epsilon' \times d_{num}$-differential privacy according to the sequential composition property.

– Line 7 of ArbDistDP selects a cut to split using Algorithm 3, and the step also satisfies ϵ'-differential privacy.

– Line 11 of ArbDistDP selects a split value for a new numerical cut using Algorithm 4, which also satisfies ϵ'-differential privacy.

– Line 12 of ArbDistDP outputs the noisy count of each leaf node (equivalence group) of the partition tree using Algorithm 5 and guarantees $\epsilon/2$-differential privacy.

– The rest of the lines of ArbDistDP are not affected when adding/removing a single record to/from the raw data; thus, these steps do not violate differential privacy.

Each non-deterministic step of ArbDistDP is differentially private, and the total privacy budget is not greater than ϵ. Hence, ArbDistDP satisfies ϵ-differential privacy according to the sequential composition property. □

Theorem 2 (Security)**.** *ArbDistDP satisfies secure two-party computation.*

Proof. The security of ArbDistDP depends on all the steps in which two parties exchange information. Since Algorithms 2, 3, and 4 are sub-algorithms of ArbDistDP, we prove the security of ArbDistDP by proving the security of Algorithms 2, 3, and 4.

– In Algorithm 2, the steps in which two parties exchange their information are only Lines 3 and 9. In Line 3, the SCP that securely compares two integers has been proven to be secure [26]. In Line 9, both parties share their values of $|D_c^{cls}| - R_i$ (where R_i is a random number) with each other, rather than the real value of $|D_c^{cls}|$. This step also is secure. Thus, Algorithm 2 satisfies secure two-party computation.

– In Algorithm 3, the steps in which the two parties communicate with each other are Lines 3, 5, and 8. The SDPP used in Line 3 and the RVP used in Line 5 have been proven to be secure in [29] and [28], respectively. The security of SCP in Line 8 has been mentioned above. Thus, Algorithm 3 satisfies secure two-party computation.

– In Algorithm 4, the steps in which the two parties exchange their information are Lines 5, 8, 11, 16, 18, 20, and 21. The SCP used in Line 5 is secure. Note that in Lines 8, 11, and 18, P1 (or P2) sends a variable named *tmp* to P2 (or P1) instead of the real value; hence, these steps are secure without revealing real information. Lines 16 and 20 are secure according to the security of Algorithm 2. Similarly, Line 21 also is secure according to the security of Algorithm 3. Thus, Algorithm 4 satisfies secure two-party computation.

In summary, ArbDistDP satisfies secure two-party computation because of the composition theorem [33]. □

Theorem 3 (Complexity). *The computation and communication complexity of ArbDistDP are bounded by $O(hnL^2 + hnK^2) + O(2^h n)$ and $O(hnL^2 + hnK) + O(2^h ne)$, respectively, where h is the number of specializations, n is the size of the integrated anonymous data, L is the bit length of operands used in the SCP, K is the security parameter of the encryption scheme used in the RCP and SSDP, and e is the bit length of an encrypted item.*

Proof. The computation and communication complexity of ArbDistDP depend on the secure computation protocols used between two parties. The protocols instrumented inside ArbDistDP are SCP, RVP, and SSDP.

- The computation and communication complexity of SCP proposed in [26] are both $O(L^2)$.
- The computation and communication complexity of RVP proposed in [28] are both $O(K^2)$.
- The computation and communication complexity of SSDP proposed in [29] are $O(K^2)$ and $O(K)$.

Let the maximum values of $|Children(v)|$ and $|\cup Cut|$ be m_1 and m_2, respectively. Thus, the computation complexity of Algorithms 2, 3, and 4 are $O(m_1 L^2)$, $O(m_2 K^2 + m_2 L^2)$, and $O(nL^2 + nK^2)$, respectively. And the communication costs for Algorithms 2, 3, and 4 are $O(m_1 d^2)$, $O(m_2 K + m_2 L^2)$, and $O(nL^2 + nK)$, respectively.

The number of leaf nodes is 2^h, and the computation and communication complexity of adding noise to leaf nodes are $O(2^h n)$ and $O(2^h ne)$, respectively.

In summary, as both parties execute h specializations, the total computation and communication costs of ArbDistDP are $O(hm_1L^2) + O(hm_2K^2 + hm_2L^2) + O(hnL^2 + hnK^2) + O(2^h n)$ and $O(hm_1L^2) + O(hm_2K + hm_2L^2) + O(hnL^2 + hnK) + O(2^h ne)$, respectively. Because $n \gg m_1$ and $n \gg m_2$, we limit the computation and communication complexity of ArbDistDP to $O(hnL^2 + hnK^2) + O(2^h n)$ and $O(hnL^2 + hnK) + O(2^h ne)$, respectively. $\square$

5. Experimental evaluation

In this section, we evaluate the performance of ArbDistDP. First, we study the effect of privacy budgets and scaling operations on the utility of the integrated anonymous data generated by ArbDistDP. Second, we compare ArbDistDP with two single-party anonymization algorithms. Third, we estimate the scalability of ArbDistDP.

All experiments were performed on a PC with a 3.4 GHz @Intel core i7 CPU and 16 GB of RAM running Windows 10 (64-bit). Each result presented below is the average over 5 runs.

5.1. Datasets and metrics

Two publicly available datasets, i.e., *Adult* and *Nursery*, are used in our experiments. The *Adult*[3] dataset is a *de facto* benchmark for testing the performance of anonymization algorithms [21, 30, 34, 35, 36, 37, 38]. It contains 45,222 census records with 8 categorical attributes, 6 numerical attributes, and a *Class* attribute representing two kinds of income levels, i.e., $\leq$*50K*, and $>$*50K*. The second dataset, *Nursery*[4], consists of personal information of individuals who apply for admission to a nursery school. It contains 12,960 records with

[3]https://archive.ics.uci.edu/ml/datasets/adult
[4]https://archive.ics.uci.edu/ml/datasets/nursery

Table 4. Details of two datasets

Datasets	Records	Attributes
Adult	45,222 ($\leq$*50K*: 34,014 $>$*50K*: 11,208)	14 (CA: 8 NA: 6)*
Nursery	12,960 (*not_recom*: 4,320 *priority*: 4,266 *spec_prior*: 4,044 *very_recom*: 328 *recommend*: 2)	8 (CA: 8)

* CA: categorical attributes NA: numerical attributes

8 categorical attributes, and a *Class* attribute representing five categories, including *not-recom*, *priority*, *spec-prior*, *very-recom*, and *recommend*. Details of the two datasets are presented in Table 4.

For the classification analysis, we randomly divide each dataset into two subsets, i.e., a training dataset and a testing dataset. To construct the scenario of arbitrarily partitioned data, we randomly select arbitrary attributes from each of the raw records to be held by P1 and let the remaining attributes be held by P2. We apply ArbDistDP to the training dataset to obtain a $\cup Cut$ and apply the $\cup Cut$ to the testing dataset to produce a generalized testing dataset. We then build a classifier on the generalized training dataset. The metrics for measuring the utility of the generalized testing dataset are defined as follows:

– Classification accuracy(CA): the classification accuracy on the generalized testing dataset;

– BA-CA: the cost of achieving a given ϵ-differential privacy requirement, where BA is the abbreviated form of baseline accuracy that is the classification accuracy measured on the raw dataset without any anonymization;

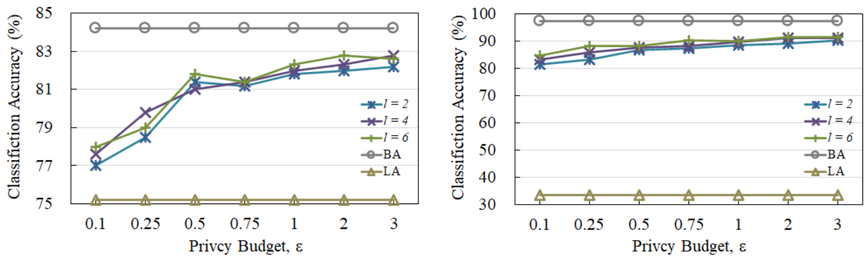
– CA-LA: the benefit of an algorithm over the random guessing, where LA is the abbreviated form of lower-bound accuracy that is the classification accuracy on the raw dataset with all attributes (except for the *Class* attribute) removed.

Note that CA is in the range of 0 to 1. The larger the value of CA is, the higher the utility of the generalized dataset is. The decision tree with default parameters in RapidMiner Studio was adopted for the classification model. The number of specializations, h, was set to 10 for ArbDistDP.

5.2. Experimental results

Data utility over different parameters. Fig. 7 shows the classification accuracy of ArbDistDP on *Adult* and *Nursery*, where the privacy budget $0.1 \leq \epsilon \leq 3$, and the scaling parameter $2 \leq l \leq 6$. It can be seen from the figure that the CA of all differentially private cases increased as ϵ increased. This occurred because a higher ϵ resulted in better attribute partitioning, and it reduced the magnitude of noise added to the number of records in each equivalence group. More specifically, in Fig. 7a BA and LA were 84.2% and 75.2%, respectively. For $\epsilon = 0.1$ and $l = 2$, BA-CA was 7.2% whereas CA-LA was 1.8%. As ϵ increased to 3, CA increased to around 82.8%, the cost decreased to about 1.4%, and the benefit increased to about 7.6%. In Fig. 7b, BA and LA were 97.3% and 33.3%, respectively. For $\epsilon = 0.1$ and $l = 2$, BA-CA was 15.8% whereas CA-LA was 48.2%. As ϵ increased to 3, CA increased to around 91.5%, the cost decreased to about 5.8%, and the benefit increased to about 58.2%. There was another trend of the CA affected by scaling parameter l. The largest span of the CA in Fig. 7a was from 78.5% to 79.8% when $\epsilon = 0.25$ and l varied from 2 to 4. The rest of CA for different values of l were close to each other if ϵ was fixed. Fig. 7b shows the similar trends of the CA for *Nursery*, with the only difference being in the case of the values of ϵ and l when getting the largest span. These results mean that the scaling operation had limited impact on the data utility.

Data utility over different algorithms. We compared ArbDistDP with DiffGen [14], by which our work is inspired. Both algorithms are combined with the generalization technique with output perturbation to mask raw data, but DiffGen only handles centralized data of a single party. We also compared ArbDistDP with another single-party algorithm, DiffP-C4.5 [21], which is an interactive algorithm for building a classifier. We set $l = 2$ for ArbDistDP. Fig. 8

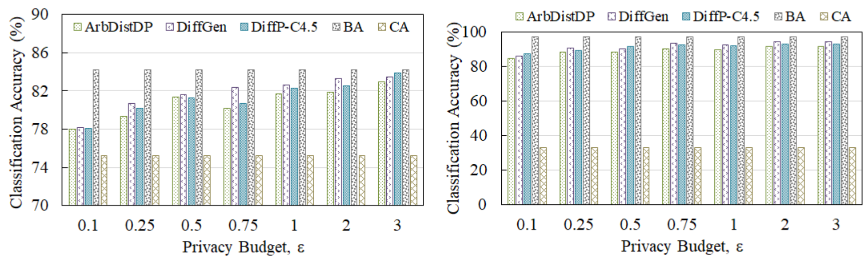


(a) *Adult* (b) *Nursery*

Fig. 7. Classification accuracy over different parameters

shows the results of DiffP-C4.5, DiffGen, and ArbDistDP on *Adult* and *Nursery*. In Fig 8a, when $\epsilon = 0.75$, the CA of DiffGen and ArbDistDP was 82.4% and 80.2%, respectively. The difference of the two values is 2.2%, which is the largest difference between the accuracy of DiffGen and ArbDistDP. When $\epsilon = 0.25$, the CA of DiffP-C4.5 and ArbDistDP was 80.2% and 79.3%, respectively. The difference of the two values is 0.9%, which is the largest difference between the accuracy of DiffP-C4.5 and ArbDistDP. In Fig 8b, when $\epsilon = 0.75$, the CA of DiffGen and ArbDistDP was 93.4% and 90.2%, respectively. The difference of the two values is 3.2%, which is the largest difference between the accuracy of DiffGen and ArbDistDP. When $\epsilon = 0.1$, the CA of DiffP-C4.5 and ArbDistDP was 87.7% and 84.6%, respectively. The difference of the two values is 3.1%, which is the largest difference between the accuracy of DiffP-C4.5 and ArbDistDP. It is worth noting that there was a gap between the CA of ArbDistDP and the BA in all differentially private cases. This is because the generalization of the raw data resulted in certain losses of information. When the value of ϵ became larger, the loss of utility might be reduced but not avoided. However, such a gap still existed in the cases of DiffP-C4.5 and ArbDistDP. Thus, it can be claimed from these results that ArbDistDP achieved comparable utility to the single-party algorithms. Also, the advantage of ArbDistDP is that it can handle arbitrarily partitioned data between two parties.

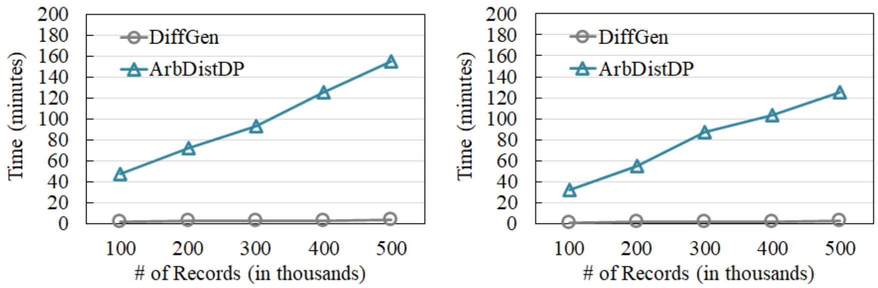
Scalability. Fig. 9 shows the running time of ArbDistDP and DiffGen on



(a) *Adult*   (b) *Nursery*

Fig. 8. Comparison of ArbDistDP with DiffGen and DiffP-C4.5

Adult and *Nursery* with 100,000 to 500,000 data records. We generated multiple versions of *Adult* and *Nursery* by randomly duplicating their records. We fixed $\epsilon = 1$ for ArbDistDP and DiffGen and set $l = 2$ for ArbDistDP. Fig. 9 shows that DiffGen was more efficient than ArbDistDP in terms of runtime. DiffGen only deals with centralized data and does not consider any exchange of information between different parties. By contrast, most processing time of ArbDistDP was spent by exchange information during the specialization. For example, in the distributed exponential mechanism, the computation cost of a modular exponentiation was roughly equal to that of an oblivious transfer protocol. For better privacy protection, when searching for split values for the numerical attributes, ArbDistDP calculates the utility scores of all possible values in the numerical domains. However, this operation results in more running time. For example, it is obvious that in Fig. 9 the time spent on *Adult* was much more than that time spent on *Nursery* since *Nursery* only contains categorical attributes and does not need to spend time selecting split values for numerical attributes. To accelerate the running speed of ArbDistDP, we maintain and update information of new candidates in $\cup Cut$, which is required by each utility score calculation, instead of repeatedly scanning all data records.



(a) *Adult*

(b) *Nursery*

Fig. 9. Scalability on *Adult* and *nursery*

6. Related work

Relational data anonymization. Research on relational data anonymization started with Samarati [7] and Sweeney [8]. They formalized a model called k-anonymity to resist record linkage attacks by generalizing or suppressing certain identifying attributes. Many variants of the model have been proposed for preserving data privacy further. Amiri et al. [34] proposed algorithms to generate k-anonymous β-likeness data that prevent identity and attribute disclosures and hide the correlations between identifying attributes and sensitive attributes. Zhu et al. [35] presented an independent l-diversity principle and its implementation to prevent corruption attacks while maintaining the utility of published data. Wang et al. [36] used the t-closeness model [9] to protect the privacy of multiple sensitive attributes. Agarwal et al. [37] proposed a (P, U)-sensitive k-anonymity model for protecting sensitive records rather than protecting sensitive attributes. Unfortunately, these works fail to provide rigorous privacy guarantees because their underlying privacy models rely on the limitations of adversaries' prior knowledge.

Differential privacy. Since this paper focuses on the non-interactive setting in which the perturbed data are released once by data publishers, only related work in such a setting is reviewed here. Mohammed et al. [14] generalized raw data to equivalence groups in a differentially private manner and added noise to the real number of records within each group. Li et al. [39] proposed a differentially private data publishing approach using cell merging. Their proposed approach consists of two sub-modules, one for partitioning the data space and the other for merging adjacent data cells with similar density. Soria-Comas et al. [40] presented an approach to generate differentially private data by adding noise to the microaggregated version of the raw data. They focused on the microaggregated data as the target of protection, instead of the raw data. Piao et al. [18, 41] successively studied the risk of citizens' privacy disclosure related to governmental data publishing. They proposed a differentially private framework for publishing governmental statistical data using fog computing. Sun et al. [19] presented two approaches to release medical data under differential privacy. They calculated attribute weights via a decision tree and used these weights to influence the degree of noise that was added to attributes. However, these works focus on centralized data of a single party, and they cannot easily be extended to handle partitioned data because the secure protocols for communication between different parties must be designed elaborately.

Secure distributed data publishing. We group the ways of data partitioning into three main categories: horizontal partitioning, vertical partitioning, and arbitrary partitioning. (1) *Horizontal partitioning.* Hasan et al. [38] proposed an approach to prevent composition attack for multiple independent data publications. They adopted the slicing technique to increase the probability of false matches between quasi-identifying values and sensitive values. The publishing scenario they defined can be converted into the scenario of horizontal data partitioning. Cheng et al. [42] studied the problem of releasing horizontally partitioned, high-dimensional data under differential privacy. They let data owners and a semi-trusted curator collaboratively build a Bayesian network for data sharing. (2) *Vertical partitioning.* Tang et al. [43] presented a differentially private approach for publishing vertically partitioned data. Their approach also involves an intermediary. However, it may be unsafe for data owners to integrate their data with an external party, and it increases the cost of communication between participants. Soria-Comas et al. [44] presented two protocols for vertical data anonymization based on the observation that there is a clear separation between quasi-identifying attributes and sensitive attributes. The difference between the two protocols lies on the attributes that are masked to preserve privacy. Sharma et al. [45] proposed a secure model to preserve the privacy of vertically partitioned data. They generated arbitrary cryptographic keys and used these keys to encrypt attribute values for preserving privacy. Wimmer et al. [46] proposed a multi-agent system to integrate distributed medical data. Their system consists of different types of agents, each of them combined some anonymization techniques. The system can be adapted to both scenarios of horizontal and vertical partitioning. Some privacy-preserving data publishing approaches [30, 47, 48, 49] also were proposed in the distributed setting. There is little work that concentrates on arbitrarily partitioning data. To the best of our knowledge, we take the first step to deal with differentially private data publishing for arbitrarily partitioned data in the literature.

7. Conclusions

We propose a differentially private algorithm for anonymizing arbitrarily partitioned data between two parties in the semi-honest model. The proposed algorithm uses a series of secure protocols to guide the collaborative anonymization; to guarantee differential privacy, it generalizes attribute values in a probabilistic manner and adds Laplacian noise to the generalized result. The experimental results demonstrated that the proposed algorithm achieved good classification accuracy while preserving data privacy and provided similar data utility when compared to two single-party approaches.

We have planned several directions for our future work. First, an extension of integrating and masking arbitrarily partitioned data among three or more parties is worth considering. The primary challenge is that secure protocols instrumented in our proposed algorithm must be redesigned to securely exchange information among multiple parties, and the computation and communication cost of these new protocols should be acceptable. Second, we will focus on collaborative anonymization for other data analysis, such as cluster analysis. The key challenge is to design a utility function with low sensitivity that not only works for the specified data analysis but also can be compatible with certain implementation mechanisms of differential privacy.

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